UNIT-III

Reasoning with Uncertain Information:

- An agent has only uncertain information about its task and about its environment.
- ➤ A statement such as P∨Q allows us to express uncertainty about which of P or Q is true.
- ➤ How certain we are about P or Q.
- \triangleright We can deduce Q from P and P \supset Q. that is if an agent knows P \supset Q and it subsequently learns P, it can infer Q also.
- ➤ Are there analogous inference processes when information is uncertain?
- ➤ Various formalisms have been employed to represent and reason about uncertain information.
- The formalism that is most well developed is based on probabilities.

3.1 Review of Probability Theory:

The primitives in probabilistic reasoning are *random variables*. Just like primitives in Propositional Logic are propositions.

A random variable is not in fact a variable, but a function from a sample space S to another space, often the real numbers.

For example, let the random variable Sum (representing outcome of two die throws) be defined thus:

Sum (die1, die2) = die1 + die2

Each random variable has an associated probability distribution determined by the underlying distribution on the sample space

Continuing our example: P(Sum = 2) = 1/36,

P(Sum = 3) = 2/36, ..., P(Sum = 12) = 1/36

Consdier the probabilistic model of the fictitious medical expert system mentioned before. The sample space is described by 8 binary valued variables.

Visit to Asia? A

Tuberculosis? T

Either tub. or lung cancer? E

Lung cancer? L

Smoking? S

Bronchitis? B

Dyspnoea? D

There are $2^8 = 256$ events in the sample space. Each event is determined by a joint instantiation of all of the variables.

$$S = \{(A = f, T = f, E = f, L = f, S = f, B = f, D = f, X = f), (A = f, T = f, E = f, L = f, S = f, B = f, D = f, X = t), ... (A = t, T = t, E = t, L = t, S = t, B = t, D = t, X = t)\}$$

Since S is defined in terms of joint instantations, any distribution defined on it is called a joint distribution. Il underlying distributions will be joint distributions in this module. The variables {A, T, E, L, S, B, D, X} are in fact random variables, which 'project' values.

$$L(A = f, T = f, E = f, L = f, S = f, B = f, D = f, X = f) = f$$

 $L(A = f, T = f, E = f, L = f, S = f, B = f, D = f, X = t) = f$
 $L(A = t, T = t, E = t, L = t, S = t, B = t, D = t, X = t) = t$

Each of the random variables $\{A,T,E,L,S,B,D,X\}$ has its own distribution, determined by the underlying joint distribution. This is known as the margin distribution. For example, the distribution for L is denoted P(L), and this distribution is defined by the two probabilities P(L = f) and P(L = t). For example,

$$P(L = f)$$

= $P(A = f, T = f,E = f,L = f, S = f,B = f,D = f,X = f)$
+ $P(A = f, T = f,E = f,L = f, S = f,B = f,D = f,X = t)$
+ $P(A = f, T = f,E = f,L = f, S = f,B = f,D = t,X = f)$
...

P(L) is an example of a marginal distribution. Here's a joint distribution over two binary value variables A and B

	A=0	A=1
B=0	0.2	0.3
B=1	0.4	0.1

We get the marginal distribution over B by simply adding up the different possible values of A for any value of B (and put the result in the "margin").

	A=0	A=1	
B=0	0.2	0.3	0.5 (= 0.2 + 0.3)
B=1	0.4	0.1	0.5 (=0.4 + 0.1)

In general, given a joint distribution over a set of variables, we can get the marginal distribution over a subset by simply summing out those variables not in the subset.

In the medical expert system case, we can get the marginal distribution over, say, A,D by simply summing out the other variables:

$$P(A,D) = \sum_{T,E,L,S,B,X} P(A,T,E,L,S,B,D,X)$$

However, computing marginals is not an easy task always. For example,

$$P(A = t, D = f)$$

$$= P(A = t, T = f, E = f, L = f, S = f, B = f, D = f, X = f)$$

$$+ P(A = t, T = f, E = f, L = f, S = f, B = f, D = f, X = t)$$

$$+ P(A = t, T = f, E = f, L = f, S = f, B = t, D = f, X = f)$$

$$+ P(A = t, T = f,E = f,L = f, S = f,B = t,D = f,X = t)$$

. . .

$$P(A = t, T = t, E = t, L = t, S = t, B = t, D = f, X = t)$$

This has 64 summands! Each of whose value needs to be estimated from empirical data. For the estimates to be of good quality, each of the instances that appear in the summands should appear sufficiently large number of times in the empirical data. Often such a large amount of data is not available.

However, computation can be simplified for certain special but common conditions. This is the condition of independence of variables.

Two random variables A and B are independent iff

$$P(A,B) = P(A)P(B)$$

i.e. can get the joint from the marginals

This is quite a strong statement: It means for any value x of A and any value y of B

$$P(A = x,B = y) = P(A = x)P(B = y)$$

Note that the independence of two random variables is a property of a the underlying probability distribution. We can have

Conditional probability is defined as:

$$P(A|B) \stackrel{\text{def}}{=} \frac{P(A,B)}{P(B)}$$

It means for any value x of A and any value y of B

$$P(A = x | B = y) = \frac{P(A = x, B = y)}{P(B = y)}$$

If A and B are independent then

$$P(A|B) = P(A)$$

Conditional probabilities can represent causal relationships in both directions. From cause to (probable) effects

$$Car_start = f \leftarrow Cold_battery = t$$

 $P(Car_start = f \mid Cold_battery = t) = 0.8$

From effect to (probable) cause

$$Cold_battery = t \leftarrow Car_start = f$$

 $P(Cold_battery = t \mid Car_start = f) = 0.7$

3.2 Probabilistic Inference Rules:

Two rules in probability theory are important for inferencing, namely, the product rule and the Bayes' rule.

Product rule:

$$P(A,B|C) = P(A|B,C)P(B|C)$$
$$= P(B|A,C)P(A|C)$$

Bayes' rule:

$$P(A|B,C) = \frac{P(B|A,C)P(A|C)}{P(B|C)}$$

Used in Bayesian statistics:

$$P(Model|Data) = \frac{P(Model)P(Data|Model)}{P(Data)}$$

Here is a simple example, of application of Bayes' rule.

Suppose you have been tested positive for a disease; what is the probability that you actually have the disease?

It depends on the accuracy and sensitivity of the test, and on the background (prior) probability of the disease.

Let $P(Test=+ve \mid Disease=true) = 0.95$, so the false negative rate,

P(Test=-ve | Disease=true), is 5%.

Let $P(Test=+ve \mid Disease=false) = 0.05$, so the false positive rate is also 5%.

Suppose the disease is rare: P(Disease=true) = 0.01 (1%).

Let D denote Disease and "T=+ve" denote the positive Tes.

Then,

So the probability of having the disease given that you tested positive is just 16%. This seems too low, but here is an intuitive argument to support it. Of 100 people, we expect only 1 to have the disease, but we expect about 5% of those (5 people) to test positive.

So of the 6 people who test positive, we only expect 1 of them to actually have the disease; and indeed 1/6 is approximately 0.16.

In other words, the reason the number is so small is that you believed that this is a rare disease; the test has made it 16 times more likely you have the disease, but it is still unlikely in absolute terms. If you want to be "objective", you can set the prior to uniform (i.e. effectively ignore the prior), and then get

$$P(D=true|T=+ve) = \frac{P(T=+ve|D=true) * P(D=true)}{P(T=+ve)}$$

$$= \frac{0.95 * 0.5}{0.95 * 0.5} = \frac{0.475}{0.95 * 0.5} = 0.95$$

This, of course, is just the true positive rate of the test. However, this conclusion relies on your belief that, if you did not conduct the test, half the people in the world have the disease, which does not seem reasonable.

A better approach is to use a plausible prior (eg P(D=true)=0.01), but then conduct multiple independent tests; if they all show up positive, then the posterior will increase. For example, if we conduct two (conditionally independent) tests T1, T2 with the same reliability, and they are both positive, we get

The assumption that the pieces of evidence are conditionally independent is called the **naive Bayes** assumption. This model has been successfully used for mainly application including classifying email as spam (D=true) or not (D=false) given the presence of various key words (Ti=+ve if word i is in the text, else Ti=-ve). It is clear that the words are not independent, even conditioned on spam/not-spam, but the model works surprisingly well nonetheless.

In many problems, complete independence of variables do not exist. Though many of them are conditionally independent.

X and Y are conditionally independent given Z iff

$$P(X,Y|Z) = P(X|Z)P(Y|Z)$$

In full: X and Y are conditionally independent given Z iff for any instantiation x, y, z of X, Y,Z we have

$$P(X = x, Y = y|Z = z)$$

$$= P(X = x|Z = z)P(Y = y|Z = z)$$

An example of conditional independence:

Consider the following three Boolean random variables:

LeaveBy8, GetTrain, OnTime

Suppose we can assume that:

 $P(OnTime \mid GetTrain, LeaveBy8) = P(OnTime \mid GetTrain)$

but NOT $P(OnTime \mid LeaveBy8) = P(OnTime)$

Then, OnTime is dependent on LeaveBy8, but independent of LeaveBy8 given GetTrain.

We can represent $P(OnTime \mid GetTrain, LeaveBy8) = P(OnTime \mid GetTrain)$

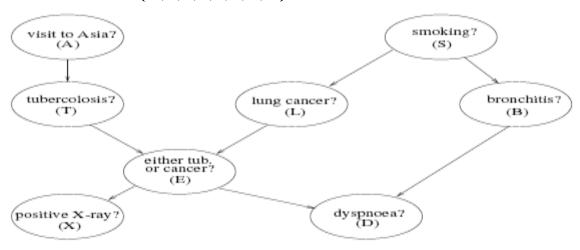
graphically by: LeaveBy8 -> GetTrain -> OnTime

3.3 Bayesian Networks:

3.3.1 Representation and Syntax

Bayes Nets (BN) (also referred to as Probabilistic Graphical Models and Bayesian Belief Networks) are directed acyclic graphs (DAGs) where each node represents a random variable. The intuitive meaning of an arrow from a parent to a child is that the parent directly influences the child. These influences are quantified by conditional probabilities.

BNs are graphical representations of joint distributions. The BN for the medical expert system mentioned previously represents a joint distribution over 8 binary random variables {A,T,E,L,S,B,D,X}.



3.3.2 Conditional Probability Tables

Each node in a Bayesian net has an associated conditional probability table or CPT. (Assume all random variables have only a finite number of possible values). This gives the probability values for the random variable at the node conditional on values for its parents.

Here is a part of one of the CPTs from the medical expert system network.

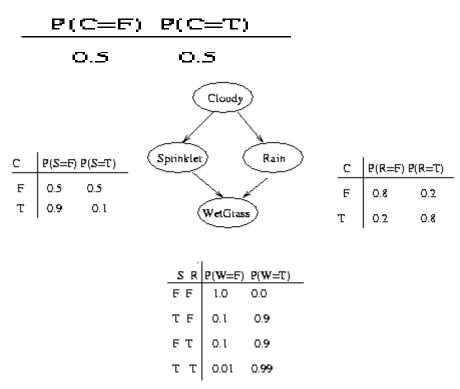
$$P(D = t|E = t, B = t) = 0.9$$
 $P(D = t|E = t, B = f) = 0.7$ $P(D = t|E = f, B = t) = 0.8$ $P(D = t|E = f, B = f) = 0.1$

If a node has no parents, then the CPT reduces to a table giving the marginal distribution on that random variable.

$$P(A = t) = 0.1$$

 $P(A = f) = 0.9$

Consider another example, in which all nodes are binary, i.e., have two possible values, which we will denote by T (true) and F (false).



We see that the event "grass is wet" (W=true) has two possible causes: either the water sprinker is on (S=true) or it is raining (R=true). The strength of this relationship is shown in the table. For example, we see that $Pr(W=true \mid S=true, R=false) = 0.9$ (second row), and hence, $Pr(W=false \mid S=true, R=false) = 1 - 0.9 = 0.1$, since each row must sum to one. Since the C node has no parents, its CPT specifies the prior probability that it is cloudy (in this case, 0.5). (Think of C as representing the season: if it is a cloudy season, it is less likely that the sprinkler is on and more likely that the rain is on.)

3.4 Semantics of Bayesian Networks

The simplest conditional independence relationship encoded in a Bayesian network can be stated as follows: a node is independent of its ancestors given its parents, where the ancestor/parent relationship is with respect to some fixed topological ordering of the nodes.

In the sprinkler example above, by the chain rule of probability, the joint probability of all the nodes in the graph above is

$$P(C, S, R, W) = P(C) * P(S|C) * P(R|C,S) * P(W|C,S,R)$$

By using conditional independence relationships, we can rewrite this as P(C, S, R, W) = P(C) * P(S|C) * P(R|C) * P(W|S,R)

Where we were allowed to simplify the third term because R is independent of S given its parent C, and the last term because W is independent of C given its parent S and R. we can see that the conditional independence relationships allow us to represent the joint more compactly. Here the savings are minimal, but in general, if we had n binary nodes, the full joint would require O(2^n) space to represent, but the factored form would require O(n 2^k) space to represent, where k is the maximum fan-in of a node. And fewer parameters makes learning easier.

The intuitive meaning of an arrow from a parent to a child is that the parent directly influences the child. The direction of this influence is often taken to represent casual influence. The conditional probabilities give the strength of causal influence. A 0 or 1 in a CPT represents a deterministic influence.

$$P(E = t|T = t, C = t) = 1$$
 $P(E = t|T = t, L = f) = 1$ $P(E = t|T = f, L = f) = 0$

3.4.1 Decomposing Joint Distributions

A joint distribution can always be broken down into a product of conditional probabilities using repeated applications of the product rule.

$$P(A, T, E, L, S, B, D, X) = P(X|A, T, E, L, S, B, D)P(D|A, T, E, L, S, B)$$

 $P(B|A, T, E, L, S)P(S|A, T, E, L)P(L|A, T, E)P(E|A, T)P(T|A)P(A)$

We can order the variables however we like:

$$P(A, T, E, L, S, B, D, X) = P(X|A, T, E, L, S, B, D)P(D|A, T, E, L, S, B)$$

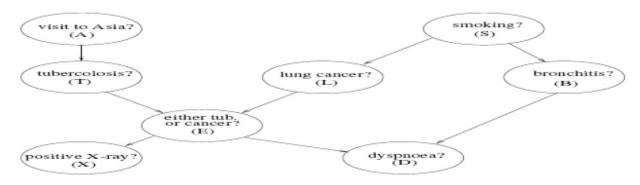
$$P(E|A, T, L, S, B)P(B|A, T, L, S)P(L|A, T, S)P(S|A, T)P(T|A)P(A)$$

3.4.2 Conditional Independence in Bayes Net:

A Bayes net represents the assumption that each node is conditionally independent of all its non-descendants given its parents.

So for example,

$$P(E|A,T,L,S,B) = P(E|T,L)$$



Note that, a node is NOT independent of its descendants given its parents. Generally,

$$P(E|A,T,L,S,B,X) \neq P(E|T,L)$$

3.5 situation calculus:

It provides a framework for representing change, actions and reasoning about them

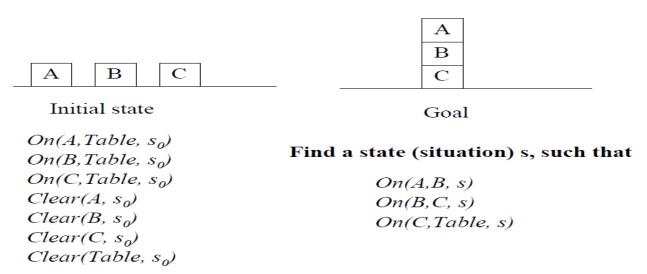
- Based on first-order logic,
- A situation variable models new states of the world
- Action objects model activities
- uses inference methods developed for FOL to do the reasoning
- Logic for reasoning about changes in the state of the world
- The world is described by:
- Sequences of **situations** of the current state
- Changes from one situation to another are caused by actions
- The situation calculus allows us to:
- Describe the initial state and a goal state
- Build the KB that describes the effect of actions (operators)
- Prove that the KB and the initial state lead to a goal state
- extracts a plan as side-effect of the proof

The language is based on the First-order logic plus:

- Special variables: s,a objects of type situation and action
- Action functions: return actions.
- E.g. Move(A, TABLE, B) represents a move action
- -Move(x,y,z) represents an action schema
- Two special function symbols of type situation
- -s0 initial situation
- -DO(a,s) denotes the situation obtained after performing an action a in situation s
- Situation-dependent functions and relations (also called fluents)
- **Relation:** On(x,y,s) object x is on object y in situation s;

- Function: Above(x,s) - object that is above x in situation s.

Situation calculus. Blocks world example.



Blocks world example.



Initial state

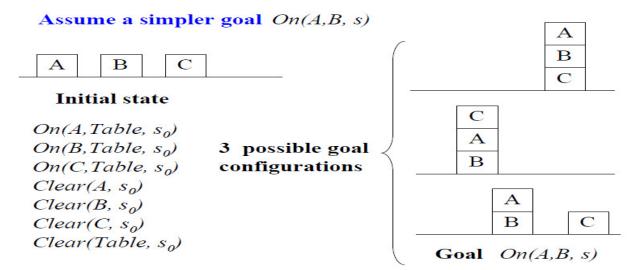
 $On(A, Table, s_0)$ $On(B, Table, s_0)$ $On(C, Table, s_0)$ $Clear(A, s_0)$ $Clear(B, s_0)$ $Clear(C, s_0)$ $Clear(Table, s_0)$

Goal

On(A,B, s) On(B,C, s)On(C,Table, s)

Note: It is not necessary that the goal describes all relations Clear(A, s)

Blocks world example.



Knowledge about the world. Axioms.

Knowledge base we need to built to support the reasoning:

• Must represent changes in the world due to actions.

Two types of axioms:

- Effect axioms
- changes in situations that result from actions
- Frame axioms
 - things preserved from the previous situation

Blocks world example. Effect axioms.

Effect axioms:

Moving x from y to z.
$$MOVE(x, y, z)$$

Effect of move changes on On relations

$$On(x, y, s) \land Clear(x, s) \land Clear(z, s) \rightarrow On(x, z, DO(MOVE(x, y, z), s))$$

$$On(x, y, s) \land Clear(x, s) \land Clear(z, s) \rightarrow \neg On(x, y, DO(MOVE(x, y, z), s))$$

Effect of move changes on Clear relations

$$On(x, y, s) \land Clear(x, s) \land Clear(z, s) \rightarrow Clear(y, DO(MOVE(x, y, z), s))$$

$$On(x, y, s) \land Clear(x, s) \land Clear(z, s) \land (z \neq Table)$$

 $\rightarrow \neg Clear(z, DO(MOVE(x, y, z), s))$

Blocks world example. Frame axioms.

- Frame axioms.
 - Represent things that remain unchanged after an action.

On relations:

$$On(u, v, s) \land (u \neq x) \land (v \neq y) \rightarrow On(u, v, DO(MOVE(x, y, z), s))$$

Clear relations:

$$Clear(u, s) \land (u \neq z) \rightarrow Clear(u, DO(MOVE(x, y, z), s))$$

Planning in situation calculus

Planning problem:

find a sequence of actions that lead to a goal

Planning in situation calculus is converted to the theorem proving problem

Goal state:

$$\exists s \ On(A,B,s) \land On(B,C,s) \land On(C,Table,s)$$

- Possible inference approaches:
 - Inference rule approach
 - Conversion to SAT
- **Plan** (solution) is a byproduct of theorem proving.
- Example: blocks world

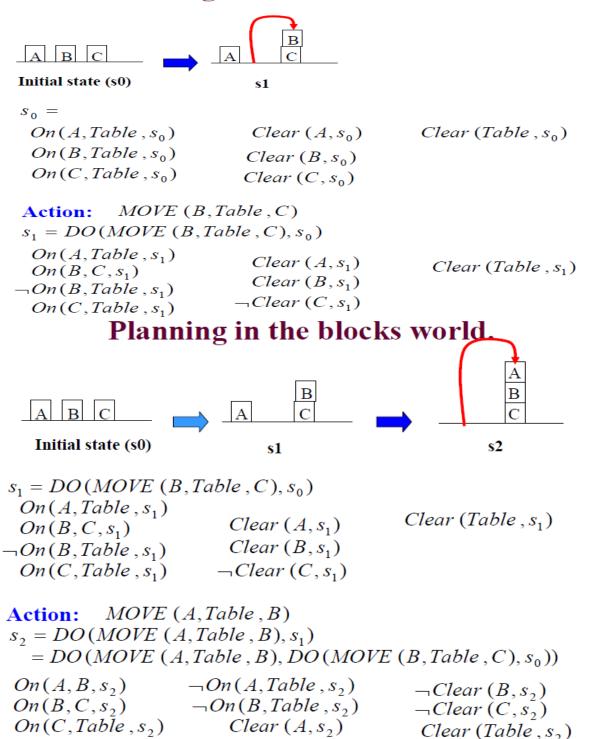
Planning in a blocks world.

Initial state	Goal
A B C	C
	В
	A

Initial state

$$On(A, Table, s_0)$$
 $On(A, B, s)$ $On(B, Table, s_0)$ $On(B, C, s)$ $On(C, Table, s_0)$ $On(C, Table, s_0)$ $On(C, Table, s_0)$ $Clear(A, s_0)$ $Clear(B, s_0)$ $Clear(C, s_0)$ $Clear(Table, s_0)$

Planning in the blocks world.



3.6 Planning in situation calculus

Planning problem:

- Find a sequence of actions that lead to a goal
- Is a special type of a search problem
- Planning in situation calculus is converted to theorem proving.

Problems:

- Large search space
- Large number of axioms to be defined for one action
- Proof may not lead to the best (shortest) plan.

Planning

• Propositional and first-order logic

- formalism for representing the knowledge about the world and ways of reasoning
- Statements about the world are true or false
- The real-world:
- is dynamic; can change over time
- an agent can actively change the world through its actions
- Planning problem: find sequence of actions that lead to a goal
- Challenges:
- Build a representation language for modeling action and change
 - Design of special search algorithms for a given representation

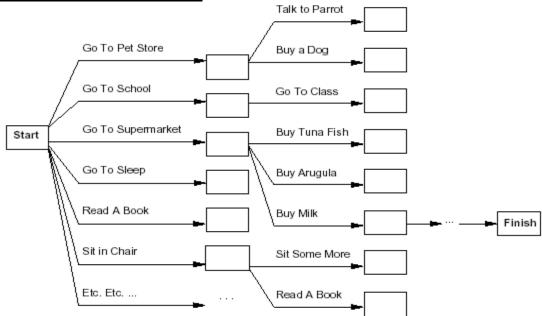
Planning and search

Planning – a special type of a search problem What if we use a standard search formulation?

Search problem:

- State space a set of states of the world among which we search for the solution.
- Initial state. A state we start from.
- Operators. Map states to new states.
- Goal condition. Test whether the goal is satisfied.
- Assume a simple problem of buying things:
 - Get a quarter of milk, bananas, cordless drill

Planning search – Example



A huge branch factor !!! Goals can take multiple steps to reach!!!

3.6.1 To address these problems planning systems:

- Open state, action and goal representations to allow selection, reasoning. Make things visible and expose the structure.
- Use FOL or its restricted subset
- Add actions to the plan sequence wherever and whenever it is needed
- Drop the need to construct solutions sequentially from the initial state
- Apply divide and conquer strategies to sub-goals when these are independent (SIMPLIFYING ASSUMPTION otherwise ...)

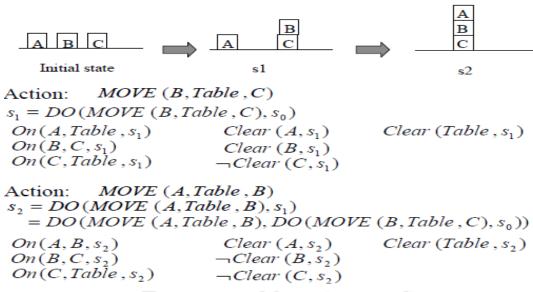
3.6.2 Planning systems. Representation.

Design of planning systems:

- Situation calculus
- based on FOL.
- a situation variable models new states of the world
- STRIPS like planners
- STRIPS STanford Research Institute Problem Solver

- Restricted language as compared to situation calculus
- Allows for more efficient planning algorithms

Inference - Plan derivation



Frame problem (general)

Frame problem

- · The need to represent a large number of frame axioms
- Solution: combine positive and negative effects in one rule

$$On(u, v, DO(MOVE(x, y, z), s)) \Leftrightarrow (\neg((u = x) \land (v = y)) \land On(u, v, s)) \lor \lor (((u = x) \land (v = z)) \land On(x, y, s) \land Clear(x, s) \land Clear(z, s))$$

Inferential frame problem:

- We still need to derive properties that remain unchanged
 Other problems:
- Qualification problem enumeration of all possibilities under which an action holds
- Ramification problem enumeration of all inferences that follow from some facts

3.7 STRIPS planner

- Restricted representation language as compared to the situation calculus
- Leads to more efficient planning algorithms:
- State-space search with structured representations of states, actions and goals

- Action representation avoids the frame problem
- STRIPS planning problem
- much like a standard search problem;

Objective: find a sequence of operators from the initial state to the goal

- States:
- conjunction of literals

On(A,B), On(B,Table), Clear(A)

represent facts that are true at a specific point in time

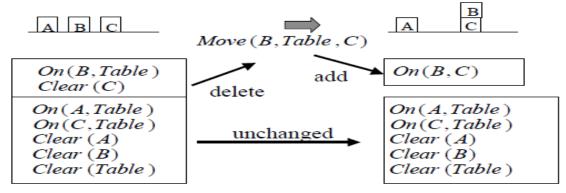
- Actions:
- Action: Move(x,y,z)
- Preconditions: conjunctions of literals with variables

On(x,y), Clear(x), Clear(z)

- **Effects.** Two lists:
- Add list: On(x,z), Clear(y)
- **Delete list:** On(x,y), Clear(z)

3.8 Forward search (goal progression)

- Main idea: Given a state s
- Unify the preconditions of some operator a with s
- Add and delete sentences from the add and delete list of an operator a from s to get a new state (can be repeated)



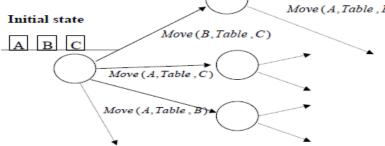
- · Use operators to generate new states to search
- Check new states whether they satisfy the goal

 Initial state

 Move (A, Table, B)

 G

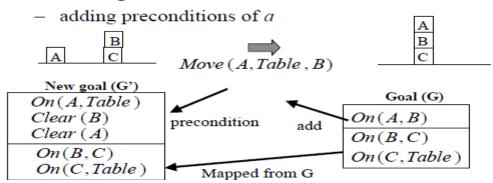
 goal



Backward search (goal regression)

Main idea: Given a goal G

- Unify the addition list of some operator a with a subset of G
- If the delete list of a does not removes elements of G, then the goal regresses to a new goal G' that is obtained from G by:
 - deleting add list of a



- · Use operators to generate new goals
- · Check whether the initial state satisfies the goal

