B.E (ECE & EEE) III Sem Internal Examination- I, A.Y- 2022-23 Set 2

Probability Theory and Stochastic Process (SBS 303 MT)

Time: 2:00pm – 3:15pm **Date: 27th Oct 2022** **Max Marks: 25**

Part – A

(Answer all Questions) **(5*2= 10 Marks)**

1. Define Normal distribution (2 M)
2. State Baye's theorem. (2 M)
3. Find the mean of Uniform distribution. (2 M)
4. If $P(A) = 0.4,\ P(B) = 0.6, P(A/B) = 0.5, then\ find\ P(B/A) and\ P(AUB).$ (2 M)
5. In a Poisson distribution if $3P(x = 2) = P(x = 4). find\ P(x = 3).$ (2 M)

Part – B

(Answer any three out of four Questions) **(3*5= 15 Marks)**

6. If x is a random variable with probability distribution function (5M)

X	0	1	2	3	4	5	6
P(x)	0.15	0.1	0.05	0.3	0.2	0.1	0

Find $E(x + 1), E(3x + 2), V(3x + 4)$

7. The content of three urns are: 1white, 2red, 3green balls; 2white, 1red, 1green balls and 4white, 5red, 3green balls. Two balls are drawn from an urn chosen at random. These are found to be one white and one green. Find the probability that the balls so drawn came from the third urn. (5M)
8. Derive the moment generating function of Normal distribution. (5M)
9. Fit a Poisson distribution for the following data (5M)

X	0	1	2	3	4
f	122	60	15	2	1

Probability Theory and Stochastic Process

[Time: 1 Hour 15 Mins] [Time: 9:30 AM – 11:00 AM] [Max. Marks: 25]

Note: 1) Answer all questions of Part A
2) Answer any three questions from Part B

PART A

1) Write the normal equations for fitting a straight line. [2M]
2) State Central limit theorem. [2M]
3) What is a wide sense stationary random process? [2M]
4) Define power density spectrum and average power density. [2M]
5) Define Random process and write the differences between Random variables and Random process. [2M]

PART B

6) Let X and Y be jointly continuous random variable with joint density function [5M]

$$f_{XY}(x,y) = \begin{cases} xy\, e^{-\left(\frac{x^2+y^2}{2}\right)}, & x>0, y>0 \\ 0, & otherwise \end{cases}$$ Check whether X and Y are independent.

7) Fit a polynomial of second degree to the data points given in the following table [5M]

X	0	1	2
Y	1	6	17

8) Calculate the PSD of the following (a) $R_{XX}(\tau) = ae^{-b|\tau|}$ (b) $R_{XX}(\tau) = ke^{-k|\tau|}$ [5M]
9) Show that $S_{YY}(w) = |H(w)|^2 S_{XX}(w)$ [5M]

B.E III Semester (Main) Examinations - January-2023

Probability Theory & Stochastic Processes

Time: 03 Hours **Max. Marks-60**

Note: i).First Question is Compulsory. Answer any Four out of remaining Six questions.
ii) Answer to each question must be written at one place only and in the same order as they occur in the Question paper.
iii) Missing data, if any, may be suitably assumed.

Part-A 6X2=12M

1. a. Define the Probability? What are the axioms of probability? [2M CO1 BTL1]

   b. Find out the probability of getting a 2 or a 5 when a die is rolled? [2M CO3 BTL1]

   c. Sate the Central Limit Theorem with an example [2M CO2 BTL2]

   d. Summarize the different measures of Central Tendency [2M CO3 BLT2]

   e. Distinguish between ensemble averages and the time averages of random process? [2M CO4 BTL2]

   f. Write the equations of wiener-Khinchine relations? [2M CO5 BTL2]

Part-B 4X12=48M

2. a) State and prove the Baye's theorem and Total probability Theorem? [8M CO1 BTL2]

   b) When two dice are thrown, determine the probabilities from axiom 3 for the following events. (i) A= $\{Sum = 7\}$ (ii) B= $\{ 8 \leq Sum \leq 11\}$ (iii) C= $\{10 < Sum\}$ (iv) $P(B \cap C)$ [4M CO1 BTL3]

3. a) Define the CDF? State and prove its properties [6M CO1 BTL4]

   b) A continuous random variable X defined by probability density function given by $f(x)=5(1-x^4)/4 \quad 0 \leq x \leq 1$. Calculate $E[X]$, $E[X^2]$ and variance. [5M CO2 BTL3]

4. a) Explain how Moment generating function generates the moments? State and prove it's properties? [7M CO2 BTL4]

   b) The joint probability density function of f(x,y) is given by $f(x,y)= 8xy \; 0 \leq x \leq 1, 0 \leq y \leq x$
      (i) Determine the marginal density of X and Y.
      (ii) Determine the conditional density functions of X and Y. Verify that whether X and Y are independent. [5M CO2 BTL4]

Code: R122132

5. a) Fit a parabola to the following data

X	-2	-1	0	1	2
Y	29	25	22	20	19

[7M CO3 BTL4]

b) Fit a second degree polynomial for the following data by the method of least Squares

x	10	12	15	23	20
y	14	17	23	25	21

[5M CO3 BTL6]

6. a) Categorize random processes into first order ,second order , wide- sense and strict-sense stationary based on their characteristics and properties.

[6M CO4 BTL4]

b) Assess whether the random process $X(t)=A\cos(\omega_0 t+\Theta)$ is wide stationary or not, where A, ω_0 are constants and Θ is a uniformly distributed random variable on the interval $(0,2\Pi)$ [6M CO4 BTL5]

7. a) Define and prove any Four properties of PSD of random process. [6M CO5 BTL4]

b) Determine the auto correlation function for the following power density spectrum:

$$S_{XX}(\omega)=\frac{157+12\omega^2}{(16+\omega^2)(9+\omega^2)}$$

[6M CO5 BTL3]