

FACULTY OF ENGINEERING
B.E. I-Year (Backlog) Examination, March/April 2021

Subject : Mathematics - II

Time: 2 hours

Max. Marks: 75

Note: Missing Data, if any, may be suitably be assumed.

PART - A

Answer any seven questions.

(7x3=21 Marks)

- 1 Find integrating factor of the differential equation $(x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0$.
- 2 Find the orthogonal trajectories of family of curves $r^2 = a^2 \cos 2\theta$.
- 3 Solve $(D^2 + 1)y = 0$.
- 4 Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$.
- 5 Define regular point and irregular singular point of the differential equation.
- 6 Show that $P_n'(1) = \frac{n(n+1)}{2}$.
- 7 Define Complementary error function. Show that $\operatorname{erfc}(x) + \operatorname{erfc}(-x) = 2$.
- 8 Write the properties of Bessel functions.
- 9 Find $L(t \sin 3t)$.
- 10 Find $L^{-1}\left(\frac{x^2 - 3x + 4}{x^3}\right)$.

PART - B

Answer any three questions.

(3x18 = 54 Marks)

- 11 (a) Solve $\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$.
 (b) A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C . What will be the temperature of the body after 40 minutes from the original.
- 12 (a) Solve by method of variation of parameters:
 $\frac{d^2y}{dx^2} + y = \tan x$.
 (b) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \sin 2x$.
- 13 (a) Solve in series of the equation $\frac{d^2y}{dx^2} + x^2y = 0$.
 (b) Show that $n P_n(x) = x P_n'(x) - P_{n-1}'(x)$.

14 (a) Show that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

(b) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where $m, n > 0$.

15 (a) Find $L\left(\int_0^{\infty} e^{-t} \cos t dt\right)$.

(b) Using convolution theorem, evaluate $L^{-1}\left(\frac{1}{(s^2+1)(s^2+9)}\right)$.

16 (a) Solve $x^2(y - px) = yp^2$.

(b) Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \log x$.

17 (a) Show that $\int_{-1}^1 x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2-1}$.

(b) Using Laplace Transform, solve the equation

$(D^2 + n^2)x = a \sin(nt + \alpha), x = Dx = 0 \text{ at } t = 0$