

Time : 3 Hours

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each Question carries 14 Marks.

(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.

(iii) Missing data, if any, may be suitably assumed.

1. (a) If λ is an eigenvalue of a non-singular matrix A , show that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1} .

(b) Obtain the general solution of the differential equation $y = xy' + e^{-y}$.

(c) Find the second order differential equation for which e^x, e^{-x} are solutions.

(d) Prove that $\operatorname{erf}(x) + \operatorname{erfc}(x) = 1$.

(e) Find $L\left\{\left(\cos t - \sin t\right)^2\right\}$.

(f) Find the matrix of the quadratic form $Q = 2(x^2 + xy + y^2)$.

(g) Find a particular integral of $y'' + 2y' + y = \sin x$.

2. (a) Show that the system of equations $x - 3y - 8z + 10 = 0$, $3x + y - 4z = 0$, $2x + 5y + 6z - 13 = 0$ is consistent and solve the same.

(b) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

3. (a) Find the general solution of $(x^2 + y^2) dx - xy^2 dy = 0$.

(b) Solve the differential equation $xy(1 + xy^2) \frac{dy}{dx} = 1$.

4. (a) Solve $\frac{d^3y}{dx^3} - y = (e^x + e^{-x})^2$.
 (b) Solve $x^2y'' - 2xy' + 2y = \frac{1}{x}$.

5. (a) Prove that $\beta(m, n) = \beta(n, m)$ and $\beta(m+1, n) + \beta(n+1, m) = \beta(m, n)$.
 (b) Find the power series solution of the differential equation $(1-x^2)y'' + 2y' = 0$ about the origin.

6. (a) Evaluate $\int_0^{\pi} t^2 e^{-t} \sin t \, dt$ using Laplace transform.

- (b) Apply convolution theorem to find $L^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}$.

7. (a) Define rank of a matrix. Find all values of k such that the rank of the matrix

$$A = \begin{bmatrix} k & 7 & 0 & 0 \\ 0 & k & -1 & 0 \\ 0 & 0 & k & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$$

is equal to 3.

- (b) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is a parameter.