Code No. E-5612/N/BL/AICTE

FACULTY OF ENGINEERING

B.E. (Common to all Branches) II Semester (AICTE) (Main & Backlog) Examination, September / October 2023

Subject: Mathematics-II

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each questions carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1. (a) If the sum of the eigen values of $A=\begin{bmatrix} 5 & 7 & 3 \\ -2 & k & 5 \\ 0 & 3 & 2 \end{bmatrix}$ is -10, then find k.

(b) Define an exact differential equation.

(c) Solve $x^2y'' - 2xy' - 4y = 0$.

(d) Evaluate $\int_0^{\pi/2} \sin^7\theta \cos^5\theta \, d\theta$ using Gamma and Beta functions.

(e) Find $L\{e^{-4t}\, t^2\}$.

(f) Find the matrix of the quadratic form

$$Q = 2x_1^2 + 4x_2^2 + 5x_3^2 - 6x_1x_2 + 8x_2x_3 - 10x_3x_1.$$

(g) Obtain the singular solution of $y = xy' - \frac{1}{y'}$.

2. (a) Find the rank of the matrix $A=\begin{bmatrix} 2 & -1 & 3 & 2 \\ -4 & 0 & 3 & 5 \\ 7 & 2 & 1 & 1 \end{bmatrix}$ by reducing to echelon form.

(b) Verify Cayley-Hamilton theorem for $A=\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and hence find A^{-1}.

3. (a) Solve $(x^2 + y^3)dx - xy^2dy = 0$.

(b) Find the orthogonal trajectories of the family of circles passing through $(0,2)$ and $(0,-2)$.

4. (a) Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 3e^{-x} + 2x + \sin x$.

(b) Solve $\frac{d^2y}{dx^2} + y = \tan x$ by the method of variation of parameters.

5. (a) Prove that $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$.

(b) State Rodrigue's formula and hence find $P_0(x), P_1(x), P_2(x)$ and $P_3(x)$.

6. (a) Find (i) $L\left\{\frac{\sinh t}{t}\right\}$ and (ii) $L\{e^{-t}\sin^2 t\}$

(b) Using Laplace transforms, solve $\frac{d^2y}{dt^2} + 25y = 10\cos 5t,\ y(0) = 2,\ y'(0) = 0$.

7. (a) Find the values of λ and μ for which the system equations $x + y + z = 3$, $x + 2y + 2z = 6$, $x + \lambda y + 3z = \mu$ has (i) no solution (ii) a unique solution and (iii) infinite number of solutions.

(b) Using convolution theorem, find $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$.

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