

FACULTY OF ENGINEERING

B.E. II Semester (AICTE) (Backlog) (Old) Examination, August/September 2024

Subject: Mathematics-II

Time: 3 Hours

Max. Marks: 70

(Missing data, if any, may be suitably assumed)

PART - A

Note: Answer all the questions.

(10 x 2 = 20 Marks)

- Find the rank of the matrix $A = \begin{bmatrix} 1 & 4 & 5 \\ 3 & 7 & 22 \\ 2 & 6 & 8 \end{bmatrix}$ using elementary row operations.
- Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.
- Find the integrating factor and hence solve the differential equation $(x^3y^2 + x)dy + (x^2y^3 - y)dx = 0$.
- Find the general and singular solution of the Clairaut's equation $y' = px + (1 + p^2)$.
- Find the general solution of the equation $y'' + 5y' + 6y = 2e^x$.
- Solve $(D^4 + 18D^2 + 81)y = 0$.
- Define Beta function and show that $\beta(m, n) = \beta(n, m)$.
- Define Gamma Function and prove that $\Gamma(\alpha + 1) = \alpha \Gamma(\alpha)$.
- Show that $L\{f'''(t)\} = s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$.
- Find the $L\{e^{2t} + 4t^3 - 2 \sin 3t + 3 \cos 3t\}$.

PART - B

Note: Answer any five questions.

(5 x 10 = 50 Marks)

- Find Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Is it diagonalizable, if so, write the modular matrix.

- a) Use Gauss-Jordan method to find the inverse of the matrix $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$.

b) Test for consistency and solve the following system of equations:

$$x + 2y + z = 3, 2x + 3y + 2z = 5, 3x - 5y + 5z = 2 \text{ and } 3x + 9y - z = 4.$$

- a) Find the orthogonal trajectory of the Cardioids $r = a(1 - \cos \theta)$.
b) Solve $y(\log y)dx + (x - \log y)dy = 0$.

14. a) $\frac{dy}{dx} + 4xy + xy^3 = 0$.

b) Solve $y'' - 2y' + y = e^x \log x$ by method of variation of parameters.

15. a) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = \sin[2 \log(1+x)]$.

b) Find power series solution about the origin of the first order differential equation $y' = xy$.

16. Solve the differential equation $(1-x^2)y'' - 2xy' + 12y = 0$ using power series about the origin.

17. a) Solve the initial value problem $y''' - 5y'' + 7y' - 3y = 0$, $y(0)=1$, $y'(0)=0$, $y''(0)=-5$.

b) Write convolution theorem and solve the differential equation

$$y'' + 3y' + 2y = e^{-t}, y(0) = 0, y'(0) = -1.$$

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