

FACULTY OF ENGINEERING

B.E. II Semester (AICTE) (Main & Backlog) (New) Examination, August/September 2024

Subject: Mathematics-II

Time: 3 Hours

Max. Marks: 70

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each question carries 14 Marks.

(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.

(iii) Missing data, if any, may be suitably assumed.

1. a) Find the rank of the matrix Find rank of the matrix $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$.

b) If 1, 2, -1 are the eigen values of matrix A, Find trace of matrix $B = A - A^{-1} + A^2$.

c) Define exact differential equation and solve $e^x(\cos y \, dx - \sin y \, dy) = 0$.

d) Find the orthogonal trajectories of the family of curves $y = ce^x$, c is parameter.

e) Solve $y'' + 2y' + 2y = 0$.

f) Evaluate $\int_0^{\pi/2} e^{2t} \cos^2 t \, dt$.

g) Express the polynomial $3x^2 + 5x - 6$ in terms of Legendre polynomials.

2. a) Test for consistency and solve $4x - 3y - 9z + 6w = 0$, $2x + 3y + 3z + 6w = 6$,
 $4x - 21y - 39z - 6w = -24$.

b) Verify Cayley- Hamilton theorem and find A^{-1} where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & -1 \\ -2 & -1 & 1 \end{bmatrix}$.

3. a) Solve $\frac{dy}{dx} - y = y^2(\sin x + \cos x)$.

b) Solve the differential equation $(3x^2y^3e^y + y^3 + y^2)dx + (x^3y^3e^y - xy)dy = 0$.

4. a) Solve $(8D^2 - 14D + 5)y = 16\sin x$.

b) Solve $(D^2 + 4D + 4)y = e^{-2x}\sin x$ using method of variation of parameters.

5. a) Evaluate $\int_0^{\infty} 2^{-9x^2} \, dx$, using gamma function.

b) Find the power series solution about $x = 2$ of the differential equation

$$4y'' - 4y' + y = 0, \quad y(2) = 0, \quad y'(2) = 1/e.$$

6. a) Evaluate $\int_0^{\infty} t e^{-3t} \sin t \, dt$, using Laplace transform.

b) Using Laplace transform method, solve $y''' + 2y'' - y' - 2y = 0$, $y(0) = y'(0) = 0$, $y''(0) = 6$.

7. a) Solve $2x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} - 3y = x^3$.

b) Evaluate inverse Laplace transform of $\log \left(\frac{s+1}{s-1} \right)$.
