

UNIT-I(MATRIX)

1. Find the rank of the Matrix

$$\begin{aligned}
 (1) & \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 5 & 15 & 20 & 9 \end{bmatrix}, (2) \begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}, (3) \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}, (4) \begin{bmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \\ -4 & 5 & 12 & -1 \end{bmatrix}, (5) \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 7 & 11 & 15 & 19 \\ 9 & 15 & 21 & 57 \end{bmatrix}, (6) \begin{bmatrix} 1 & 2 & 3 & 0 \\ 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}, \\
 (7) & \begin{bmatrix} 2 & 3 & 10 & 4 \\ 3 & 12 & 7 & 1 \\ 4 & -13 & -2 & -2 \\ 5 & 4 & 3 & -15 \end{bmatrix}, (8) \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 1 & 3 & -1 \\ 8 & 13 & 14 \end{bmatrix}, (9) \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}, (10) \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{bmatrix}, (11) \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}, (12) \begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 4 \\ 4 & -1 & 7 \\ 4 & -1 & 5 \end{bmatrix}, \\
 (13) & \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}, (14) \begin{bmatrix} 2 & 0 & -1 \\ 5 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}, (15) \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 2 \\ 5 & -5 & 11 \end{bmatrix}, (16) \begin{bmatrix} 2 & 1 & -2 \\ -1 & -1 & 1 \\ 3 & 1 & -2 \end{bmatrix}, (17) \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 3 & 4 & 5 \\ 1 & 4 & -13 & -5 \end{bmatrix}, (18) \begin{bmatrix} 1 & -2 & 1 & -1 \\ 1 & 1 & -2 & 3 \\ 4 & 7 & -5 & 8 \end{bmatrix}, \\
 (19) & \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 7 & 11 & 15 & 19 \\ 9 & 15 & 21 & 27 \end{bmatrix}, (20) \begin{bmatrix} 2 & 0 & -1 & 0 \\ 4 & 1 & 0 & 5 \\ 0 & 1 & 3 & 6 \\ 6 & 1 & -2 & 6 \end{bmatrix}, (21) \begin{bmatrix} 2 & 3 & 1 & 0 & 4 \\ 3 & 1 & 2 & -1 & 1 \\ 4 & -1 & 3 & -2 & -2 \\ 5 & 4 & 3 & -1 & 5 \end{bmatrix}, (22) \begin{bmatrix} 1 & 2 & 3 & 2 \\ -1 & 1 & 3 & -5 \\ 2 & 3 & 4 & 5 \end{bmatrix}, (23) \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -1 \\ 1 & -1 & 2 \\ 5 & 4 & -2 \end{bmatrix}, (24) \begin{bmatrix} 2 & 3 & 1 & 0 & 4 \\ 3 & 1 & 2 & -1 & 1 \\ 4 & -1 & 3 & -2 & -2 \\ 5 & 4 & 3 & -1 & 5 \end{bmatrix}, \\
 (25) & \begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & -7 \\ -1 & -2 & 11 \end{bmatrix}, (26) \begin{bmatrix} 3 & -11 & 5 \\ 4 & 1 & -10 \\ 4 & 9 & -6 \end{bmatrix}, (27) \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 6 & 12 \end{bmatrix}, (28) \begin{bmatrix} 2 & -1 & -3 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & -7 & -13 & -1 \\ -1 & 5 & 9 & 1 \end{bmatrix}, (29) \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{bmatrix}, (30) \begin{bmatrix} 3 & 1 & 1 & 4 \\ 0 & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix}, \\
 (31) & \begin{bmatrix} 1 & 1 & -3 & 2 \\ 2 & -1 & -2 & -3 \\ 3 & 0 & -5 & -1 \\ 5 & -1 & -7 & -4 \end{bmatrix}, (32) \begin{bmatrix} 1 & -2 & 1 & -1 \\ 1 & 1 & -2 & 3 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix}, (33) \begin{bmatrix} 1 & 1 & -2 & -1 \\ 2 & 1 & 1 & -2 \\ 3 & 2 & -1 & -3 \\ 4 & 2 & 2 & -4 \end{bmatrix}, (34) \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}, (35) \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 4 & 4 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

2. Find all the eigen values and the corresponding eigen vectors of the Matrices and verify the Cayley-Hamilton

theorem and find A^{-1} if it exists, also find the modal matrix P which diagonalise A, if it exist, and show that $A=P^{-1}DP$.

$$\begin{aligned}
 (36) & \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, (37) \begin{bmatrix} 1 & 1 & i \\ 1 & 0 & i \\ -i & -i & 1 \end{bmatrix}, (38) \begin{bmatrix} 0 & 2 & -2 & 0 \\ 1 & 1 & 0 & -1 \\ -1 & 1 & -2 & 1 \\ -1 & 1 & -2 & 1 \end{bmatrix}, (39) \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}, (40) \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}, \\
 (41) & \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 2 & 1 \\ 4 & 3 & 1 & 2 \end{bmatrix}, (42) \begin{bmatrix} 1 & i & i \\ i & 1 & i \\ i & i & 1 \end{bmatrix}, (43) \begin{bmatrix} 8 & -6 & 2 \\ -6 & -7 & -4 \\ 2 & -4 & 3 \end{bmatrix}, (44) \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}, (45) \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \\
 (46) & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}, (47) \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}, (48) \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -2 & -2 & 0 \end{bmatrix}, (49) \begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}, (50) \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}, \\
 (51) & \begin{bmatrix} 1 & -2 & 1 \\ 2 & 3 & -2 \\ 3 & 1 & -1 \end{bmatrix}, (52) \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}, (53) \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ -1 & 3 & 4 \end{bmatrix}, (54) \begin{bmatrix} 1 & 1 & i \\ 1 & 0 & i \\ -i & -i & 1 \end{bmatrix}, (55) \begin{bmatrix} 0 & i & i \\ i & 0 & i \\ i & i & 0 \end{bmatrix}, \\
 (56) & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, (57) \begin{bmatrix} 0 & 2 & -2 & 0 \\ 1 & 1 & 0 & -1 \\ -1 & 1 & -2 & 1 \\ -1 & 1 & -2 & 1 \end{bmatrix}, (58) \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, (59) \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
 (60) & \begin{bmatrix} -3 & -2 & 1 \\ -2 & 0 & 4 \\ -1 & -3 & 5 \end{bmatrix}, (61) \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & -3 & 0 \end{bmatrix}, (62) \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}, (63) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}, (64) \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}
 \end{aligned}$$

3. Find the Matrix A whose eigen values and the corresponding eigen vectors are as given

$$(65) \ 2, 2, 4 \rightarrow (-2, 1, 0)^T, (-1, 0, 1)^T, (1, 0, 1)^T, (66) \ 1, -1, 2 \rightarrow (1, 1, 0)^T, (1, 0, 1)^T, (3, 1, 1)^T,$$

$$(67) \ 1, 2, 3 \rightarrow (1, 2, 1)^T, (2, 3, 4)^T, (1, 4, 9)^T, (68) \ 1, 1, 1 \rightarrow (-1, 1, 1)^T, (1, -1, 1)^T, (1, 1, -1)^T,$$

$$(69) \ 0, -1, 1 \rightarrow (-1, 1, 0)^T, (1, 0, -1)^T, (1, 1, 1)^T, (70) \ 0, 0, 3 \rightarrow (1, 2, -1)^T, (-2, 1, 0)^T, (3, 0, 1)^T$$

4. Determine whether the following set of vectors is Linearly independent or Linearly dependent

$$(71) \ \{(1, 1, 1)(i, i, i)(1+i, -1-i, i)\} \quad (72) \ \{(1, 1, 0, 1)(1, 1, 1, 1)(4, 4, 1, 1)(1, 0, 0, 1)\} \quad (73) \ \{(4, 2, 1)(2, 3, 2)(1, 1, 4)\}$$

$$(74) \ \{(1, 2, 3, 4)(0, 1, -1, 2)(1, 4, 1, 8)(3, 7, 8, 14)\} \quad (75) \ \{(3, 2, 7)(2, 4, 1)(1, -2, 6)\}$$

5. Reduce the following Quadratic forms into Canonical form and find rank, Index, Signature, and nature

$$(81) 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3 - 2x_3x_1, (82) 2x^2 + 5y^2 - 6z^2 - 2xy - yz + 8zx, (83) x_1^2 - (2+4i)x_1x_2 - (4-6i)x_2x_3 + x_2^2$$

$$(84) x_1^2 + 2x_2^2 + 3x_3^2 + 4x_4^2 + 2x_1x_2 + 4x_1x_3 - 6x_1x_4 - 4x_2x_3 - 8x_2x_4 + 12x_3x_4, (85) x_1^2 + 7x_2^2 + 26x_3^2 + 4x_1x_2 - 22x_2x_3 - 2x_3x_1$$

$$(86) x^2 - 4y^2 + 6z^2 + 2xy - 4xz + 2w^2 - 6zw, (87) 3x^2 + 3z^2 + 4xy + 8xz + 8yz, (88) 8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$$

$$(89) 6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1, (90) x_1^2 + 2x_1x_2 - 4x_1x_3 + 6x_2x_3 - 5x_2^2 + 4x_3^2, x_1^2 + 2ix_1x_2 - 8x_1x_3 + 4ix_2x_3 + 4x_3^2$$

$$(91) 2x_1^2 - 3x_2^2 + (6+8i)x_1x_2 + (4-2i)x_2x_3, (92) x_1^2 + 7x_2^2 + 7x_3^2 + 4x_1x_2 - 18x_2x_3 - 6x_3x_1$$

6. Using Matrix Method solve the system of equation

$$(100) 5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5, (101) 2x_1 - x_2 + x_3 = 4, x_1 + x_2 + x_3 = 1, x_1 - 3x_2 - 2x_3 = 2$$

$$(102) x - y + z = 4, 2x + y - 3z = 0, x + y + z = 2, (103) x_1 - x_2 + 3x_3 = 3, 2x_1 + 3x_2 + x_3 = 2, 3x_1 + 2x_2 + 4x_3 = 5$$

$$(104) 4x + 9y + 3z = 6, 2x + 3y + z = 2, 2x + 6y + 2z = 7, (105) -x + y + 2z = 2, 3x - y + z = 3, -x + 3y + 4z = 6$$

$$(106) 2x - z = 1, 5x + y = 7, y + 3z = 5, (107) x + 2y + 3z = 6, 2x + 4y + z = 7, 3x + 2y + 9z = 14$$

$$(108) 4x + 3y - z = 0, 3x + 4y + z = 0, x - y - 2z = 0, 5x + y - 4z = 0, (109) 3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4,$$

$$(110) 3x + 4y - z - 6t = 0, 2x + 3y + 2z - 3t = 0, 2x + y - 14z - 9t = 0, x + 3y + 13z + 3t = 0,$$

$$(111) x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6, (112) x + 2y + 3z = 1, 3x - y + z = 2, 4x + 2y + z = 3$$

$$(113) x + 2y + 3z = 1, 2x + 3y + 8z = 2, x + y + z = 3, (114) 4x + 2y - z = 9, x - y + 3z = -4, 2x + z = 1,$$

$$(115) 5x + 3y + 3z = 48, 2x + 6y - 3z = 18, 8x - 3y + 2z = 21, (116) x + y + z = 6, x - y + 2z = 5, 3x + y + z = 8,$$

$$(117) x + 2y - 3z = 1, 3x - 2y + z = 2, 4x + 2y + z = 3, (118) 9x + 4y + 3z = -1, 5x + y + 2z = 1, 7x + 3y + 4z = 1,$$

$$(119) x + y + z = 8, x - y + 2z = 6, 9x + 5y - 7z = 14, (120) 3x + 2y + 4z = 7, 2x + y + z = 4, x + 3y + 5z = 2,$$

$$(121) 4x + 3y - z = 0, 3x + 4y + z = 0, x - y - 2z = 0, 5x + y - 4z = 0, (122) 2x + 3y + 4z = 11, x + 5y + 7z = 15, 3x + 11y + 13z = 25,$$

$$(123) 4x + 3y - z = 0, 3x + 4y + z = 0, x - y - 2z = 0, 5x + y - 4z = 0, (124) 5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$$

$$(125) 4x + 3y - z = 0, 3x + 4y + z = 0, x - y - 2z = 0, 5x + y - 4z = 0, (126) 2x - 3y + z = -2, x - y + 2z = 3, 2x + y - 3z = -2,$$

$$(127) x + 4y + 7z = 1, 2x + 5y + 8z = 2, x + 2y + 3z = 1, (128) x - 4y + 7z = 8, 3x + 8y - 2z = 6, 7x - 8y + 26z = 3,$$

$$(129) x + y + z = 3, 3x - 9y + 2z = -4, 5x - 3y + 5z = 6, (130) x + y + z = 7, x + 2y + 3z = 16, x + 3y + 4z = 22,$$

$$(131) 2x - 3z = 0, 2y - 3z = 0, x - y + z = 1, (132) 5x + 3y + 14z = 4, y + 2z = 1, x + y + 2z = 0, 2x + y + 6z = 2,$$

$$(133) x - 2y + z + 2w = 1, x + y - z + w = 2, x + 7y - 5z - w = 4, (134) x + y + z + w = 4, x + y + z - w = 2, x - y + z - w = 0,$$

$$(135) -x + y + z + w = 1, x - y + z + w = 0, x + y - z + w = 0, x + y + z - w = 0, (136) 2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + z = 0,$$

$$(137) x + y + z = 6, x + 2y + 5z = 10, 2x + 3y + z = 0,$$

7. For what values of λ and μ do the equations

$$(141) x + 2y + 3z = 6, x + 3y + 5z = 9, 2x + 5y + \lambda z = \mu, (142) x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$$

$$(143) x + 2y + z = 6, x + 4y + 3z = 10, x + 4y + \lambda z = \mu$$

$$(144) 2x + 3y + 5z = 7, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu, (145) x + y + z = 6, x + 2y + 5z = 10, 2x + y + 4z = \mu,$$

$$(146) -2x + y + z = \lambda, x - 2y + z = \mu, x + y - 2z = \lambda, (147) 2x - 3y - 6z - 5t = 3, y - 4z + t = 1, 4x - 5y + 8z - 9t = \lambda$$

$$(148) x + \lambda y + 3z = 0, 4x + 3y + \lambda z = 0, 2x + y + 2z = 0, (149) 3x - 2y + z = \lambda, 5x - 8y + 9z = 3, 2x + y + \mu z = -1,$$

$$(150) x + y + 4z = 1, x + 2y - 2z = 1, \lambda x + y + z = 1, (151) \lambda x - y - z = 0, -x + \lambda y - z = 0, -x - y + \lambda z = 0,$$

$$(152) (3\lambda - 8)x + 3y + 3z = 0, 3x + (3\lambda - 8)y + 3z = 0, 3x + 3y + (3\lambda - 8)z = 0,$$

$$(153) x + y - z + t = 2, 2y + 4z + 2t = 3, x + 2y + z + 2t = \lambda (154) 2x - 5y + 2z = 8, 2x + 4y + 6z = 5, x + 2y + \lambda z = \mu,$$

$$(156) x + y + z = 1, 2x + y + 4z = \lambda, 4x + y + 10z = \lambda^2,$$

have i) no solution (ii) a Unique solution (iii) an Infinite solutions

UNIT-II(DIFFERENTIAL EQUATIONS)

FORMATION OF DIFFERENTIAL EQUATION

$$(1)y = e^x (A \cos x + B \sin x), (2)y = 9e^{3x} + 6e^{5x}, (3)y = e^{-2x} (a \cos 2x + b \sin 2x) (4)y = cx - \frac{1}{c}, cx + c^2, (e)y = \frac{a+x}{x^2+1}, (f)y = 9x^3 + 6x^2,$$

$$(5)xy = Ae^x + Be^{-x} + x^2, (6)y = ae^{2x} + be^{-3x} + ce^x (7)e^{2y} + 2axe^y + a^2 = 0$$

(8) Find the differential equation of all circles touching the axis of y , or at the origin and centres on the axis of $-X$

Variables separable form

$$(21) y - x \frac{dy}{dx} = a \left(y^2 + \frac{dy}{dx} \right), (22) 3e^x \tan y \, dx + (1 + e^x) \sec^2 y \, dy = 0, y(0) = \frac{\pi}{4} (23) (x + y)(dx - dy) = dx + dy, (24) xy \frac{dy}{dx} = 1 + x + y + xy,$$

$$(25) \frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0, (26) e^y (1+x^2) \frac{dy}{dx} = 2x(1+e^y) = 0, (27) \cos \theta \, dr - r \sin \theta \, d\theta = 0, (28) (x^2 - yx^2) \frac{dy}{dx} + (y^2 + x^2 y^2) = 0$$

$$(29) \frac{dy}{dx} = xe^{y-x^2}, (30) x\sqrt{1+y^2} \, dx + y\sqrt{1+x^2} \, dy = 0, (31) \frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

$$\text{form of } \frac{dy}{dx} = f(ax + by + c) - (32) \frac{dy}{dx} = \cos(x+y) + \sin(x+y), (33) \frac{dy}{dx} = \frac{a^2}{(x-y)^2}, (34) \frac{dy}{dx} = \sec(x+y), (35) \frac{dy}{dx} = (3x + y + 4)^2,$$

$$(36) \frac{dy}{dx} = (4x + y + 1)^2, (37) xy' = (y - x)^3, (38) y - \frac{dy}{dx} - x \tan(y - x) = 1$$

Homogeneous Equation

$$(39) x \, dy - y \, dx = \sqrt{x^2 + y^2} \, dx (40) \left[x \tan \frac{y}{x} - y \sec^2 \frac{y}{x} \right] dx + x \sec^2 \frac{y}{x} \, dy = 0, (41) \left(1 + e^y \right) dy + e^y \left[1 - \frac{x}{y} \right] dy = 0, (42) \frac{dy}{dx} = \frac{y^3 + 3x^2 y}{x^3 + 3xy^2},$$

$$(43) (x^2 + 4y^2 + xy) \, dx - x^2 \, dy = 0 (44) x(x - y) \frac{dy}{dx} = y(x + y), (45) (x^2 - y^2) \, dx = 2xy \, dy, (46) (\sqrt{xy} - x) \, dy + y \, dx = 0, (47) (x^3 + y^3) \, dy - x^2 y \, dx = 0$$

$$(48) \left[x \cos \frac{y}{x} + y \sin \frac{y}{x} \right] y - \left[y \sin \frac{y}{x} - x \cos \frac{y}{x} \right] x \frac{dy}{dx} = 0, (49) x \frac{dy}{dx} = y + x \cos^2 \frac{y}{x}, (50) \frac{dy}{dx} = \frac{y}{x} + \sin \frac{y}{x}, (51) \frac{dy}{dx} = \frac{y}{x + ye^{-2\left(\frac{x}{y}\right)}}$$

$$(52) x \, dx + \sin^2 \frac{y}{x} (y \, dx - x \, dy) = 0, (53) ye^{\frac{x}{y}} \, dx = \left(xe^{\frac{x}{y}} + y^2 \right) dy, (54) x \, dy \log \frac{x}{y} \, dx + \left[y^2 - x^2 \log \frac{x}{y} \right] dy = 0$$

Non-Homogeneous Equations

$$(55) (x + y - 1) \frac{dy}{dx} = x - y + 2, (56) \frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}, (57) (3y - 7x + 7) \, dx + (7y - 3x + 3) \, dy = 0, (58) (3y + 2x + 4) \, dx - (4x + 6y + 5) \, dy = 0$$

$$(59) (2x + y + 1) \, dx + (4x + 2y - 1) \, dy = 0, (60) \frac{dy}{dx} + \frac{2x + 3y + 1}{3x + 4y - 1} = 0, (61) (x + y)(dx - dy) = dx + dy$$

LINEAR EQUATIONS

$$\begin{aligned}(62) x(1-x^2) \frac{dy}{dx} + (2x^2-1)y &= x, (63) x^2 \frac{dy}{dx} = e^y - x, (64) \cos^2 x \frac{dy}{dx} + y = \tan x, (65) (x^2+1) \frac{dy}{dx} + 2xy = x^2, (66) (1+x^3) \frac{dy}{dx} + 6x^2 = 1+x^2, \\(67) x \log x \frac{dy}{dx} + y &= 2 \log x, (68) \frac{dy}{dx} - y \tan x = 3e^{-\sin x} y(0) = 4, (69) (1+y^2) + \left(x - e^{-\tan^{-1} y}\right) \frac{dy}{dx} = 0, (70) e^{-y} \sec^2 y \, dy = dx + x \, dy, \\(71) \cot 3x \frac{dy}{dx} - 3y &= \cos 3x + \sin 3x, (72) x^3 \sec^2 y \frac{dy}{dx} + 3x^2 \tan y = \cos x, (73) \frac{dy}{dx} + (y-1) \cos x = e^{-\sin x} \cos^2 x, (74) xy(1+xy^2) \frac{dy}{dx} = 1, \\(75) (1-x^2) \frac{dy}{dx} + xy &= y^3 \sin^{-1} x, (76) \frac{dy}{dx} + y \cot x = 4x \csc x, y\left(\frac{\pi}{2}\right) = 0, (77) \frac{dy}{dx} + y \cot x = 5e^{\cos x}, (78) \sqrt{1-y^2} \, dx = (\sin^{-1} y - x) \, dy\end{aligned}$$

Bernoulli's Equation

$$\begin{aligned}(79) \frac{dy}{dx} + \frac{x}{1-x^2} y &= x\sqrt{y}, (80) \frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y, (81) (1-x^2) \frac{dy}{dx} + xy = y^3 \sin^{-1} x, (82) 3y' - y \cos x = y^4 (\sin 2x - \cos x) \\(83) x \frac{dy}{dx} + y &= x^3 y^6, (84) x^3 \sec^2 y \frac{dy}{dx} + 3x^2 \tan y = \cos x, (85) \frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y, (86) \left(xy^2 - e^{\frac{1}{x^3}}\right) dx - x^2 y \, dy = 0, \\(87) \frac{dy}{dx} + \frac{y \log y}{x} &= \frac{y(\log y)^2}{x^2}, (88) y - \cos x \frac{dy}{dx} = y^2 (1 - \sin x) \cos x y(0) = 2, (89) \tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 y \\(90) \sec^2 y \frac{dy}{dx} + 2x \tan x &= x^3, (91) 2y \cos^2 y \frac{dy}{dx} - \frac{2}{x+1} \sin^2 y = (x+4)^3, (92) \frac{dy}{dx} = e^{x-y} (e^x - e^y), (93) \frac{dy}{dx} = (\sin x - \sin y) \frac{\cos x}{\cos y} \\(94) \frac{dy}{dx} - \frac{\tan y}{1+x} &= (1+x)e^x \sec y, (95) y \sin 2x \, dx - (1+y^2 + \cos^2 x) \, dy = 0, (96) \frac{dy}{dx} = 1 - x(y-x) - x^3(y-x)^3\end{aligned}$$

Exact differential Equation

$$\begin{aligned}(100) (hx+by+f) \, dy + (ax+hy-g) \, dx &= 0, (102) (e^y+1) \cos x \, dx + e^y \sin x \, dy = 0, (103) (xe^{xy}+2y) \frac{dy}{dx} + ye^{xy} = 0, (104) \frac{dy}{dx} + \frac{5x-17y+9}{3y-17x+15} = 0 \\(105) (r+\sin \theta - \cos \theta) \, dr + r(\sin \theta + \cos \theta) \, d\theta &= 0, (106) \frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0, (107) (2y \sin x + \cos y) \, dx = [x \sin y + 2 \cos x + \tan y] \, dy = 0, \\(108) x \, dx + y \, dy &= \frac{a^2(x \, dy - y \, dx)}{x^2 + y^2}, (109) (x^2 - 4xy - 2y^2) \, dx + (y^2 - 4xy - 2y^2) \, dy = 0, (110) y \left(x + \frac{1}{x}\right) \cos y \, dx + [x + \log x - x \sin y] \, dy = 0 \\(112) [\sec x \tan x \tan y - e^x] \, dx + \sec x \sec^2 y \, dy &= 0, (113) [\cos x \tan y + \cos(x+y)] \, dx + [\sin x \sec^2 y + \cos(x+y)] \, dy = 0, \\(114) \frac{2x}{y^3} \, dx + \frac{y^2-3x^2}{y^4} \, dy &= 0, (115) (2xy+y-\tan y) \, dx + [x^2-x \tan^2 y + \sec^2 y] \, dy = 0\end{aligned}$$

Non –Exact Homogenous Equations

$$(121)(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0, (122)(x^2 + y^2)dx = 2xydy, (123)\left(xy e^{\frac{x}{y}} + y^2\right)dx - x^2 e^{\frac{x}{y}}dy = 0, (124)(1 + xy)x dy + (1 - yx)x dx = 0$$

$$(125)r(\theta^2 + r^2)d\theta - \theta(\theta^2 + 2r^2)dr = 0, (126)(3xy^2 - y^3)dx - (2x^2y - xy^2)dy = 0, (127)y - x \frac{dy}{dx} = x + y \frac{dy}{dx},$$

$$(128)(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$$

$$y f(xy)dx + x g(xy)dy = 0 \text{ form}$$

$$(129)y(x^2y^2 + 2)dx + x(2 - 2x^2y^2)dy = 0, (130)(xy \sin xy + \cos xy)y dx + (xy \sin xy - \cos xy)x dy = 0, (131)y(1 + xy)dx + x(1 - xy)dy = 0$$

$$(132)y[xy + 2x^2y^2]dx + x[xy - x^2y^2]dy = 0, (134)(x^2y^2 + xy + 1)y dx + (x^2y^2 - xy + 1)x dy = 0$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = N f(x)$$

$$(135)2xydy - (x^2 + y^2 + 1)dx = 0, (136)(zxy + 1)y dx + (1 + 2xy - x^3y^3)dy = 0, (137)(3xy - 2ay^2)dx + (x^2 - 2axy)dy = 0$$

$$(138)(x^2 + y^2 + 2x)dx + 2ydy = 0, (139)(x^3 - 2y^2)dx + 2xydy = 0, (140)(x^2 + y^2 + x)dx + xydy = 0, (141)(x^2 + y^2)dx - 2xydy = 0$$

$$(142)\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right)dx + \frac{1}{4}(x + xy^2)dy = 0, (143)(-y + y^2)dx + xydy = 0, (144)(xy^2 - x^2)dx + (3x^2y^2 + x^2y - 2x^3 + y^2)dy = 0$$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = M g(y)$$

$$(145)(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0, (146)(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0, (147)y(2xy + e^x)dx - e^x dy = 0$$

$$(148)y(x + y + 1)dx + x(x + 3y + 2)dy = 0, (149)(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$$

Find the Orthogonal trajectories

$$(151)ay^2 = x^3, (152)xy = c^2, (153)x^2 - y^2 = c^2, (154)x^2 + 2gx + y^2 + c = 0, (156)x^2 + (y - c)^2 = c^2, (157)e^x + e^{-y} = e, (158)r = a(1 - \cos \theta) \quad ()r^n \sin n\theta = a^n$$

$$(159)r^n = a^n \cos n\theta, (160)r = \frac{2a}{1 + \cos \theta}, (161)r = 2a(\cos \theta + \sin \theta), (162)r = ce^\theta, (163)r^n \sin n\theta = a^n \quad (164)r = c(\sec \theta + \tan \theta), (165)r^2 = a^2 \cos 2\theta$$

$$(166)x^m + y^m = 9^m (\text{hypocycloids}), (167)y = ax^n, (168)y = \frac{x^3 - a^3}{3x}, (169)4ay = x^2, (170)\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1, \lambda \text{ parameter}, (171)r = \frac{2a}{1 + \cos \theta},$$

$$(172) \text{Find the equation of the family of all orthogonal trajectories of the family of circles which pass through the points } (2, 0) \text{ and } (-2, 0)$$

Self Orthogonal

$$(173)y^2 = 4a(x + a), (174)\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1, (175)\frac{x^2}{c} + \frac{y^2}{c + 2} + 1 = 0,$$

Find General solution and singular of Clairat's equations

$$\begin{aligned}
(186)(y-px)(p-1) &= p, (187)e^{4x}(p-1) + e^{2y}p^2 = 0, (188)y = (x-a)p - p^2, (189)y = xy' - (y')^3, (190)y = xy' + (y')^2, (191)p = \sin(y-xp), (192)y = xp + \frac{a}{p}, \\
(193)y &= px + \sqrt{a^2p^2 + b^2} (194)(y+px)^2 = x^2p, (195)x^2(y-px) = yp^2, (196)p^2(x^2-1) - 2pxy + y^2 - 1 = 0, (197)e^{3x}(p-1) + p^3e^{2y} = 0, (199)y = y = xp + \frac{a}{p}, \\
(200)y &= (x-a)p - p^2, (200)xp + p^2 = y, (201)y = xp + e^p y = px + \sqrt{1+p^2}
\end{aligned}$$

RiceAtis equation

$$\begin{aligned}
(206)y' &= y^2 - (2x-1)y + x^2, y = 1, (207)y' = 2xy^2 + (1-4x)y + 2x-1, y = x, (208)y' = 3y^2 - (1+6x)y + 3x^2 + x + 1, y = x \\
(209)y' &= 4xy^2 + (1-8x)y + 4x-1, y = 1
\end{aligned}$$

Solve the Differential Equations

$$\begin{aligned}
(231)\left(1+e^{\frac{x}{y}}\right)x\,dx+y\,dy &= \frac{a^2(x\,dy-y\,dx)}{x^2+y^2}, (232)dx+e^y\left(1-\frac{x}{y}\right)dy=0, (233)(y\,dx+x\,dy)x\cos\frac{y}{x}=(x\,dy-y\,dx)y\sin\frac{y}{x}, (234)\frac{dy}{dx}=\frac{y}{x}+\tan\left(\frac{y}{x}\right) \\
(235)y\sin 2x\,dx-(y^2+\cos^2 x)\,dy &= 0, (236)(x^2-ay)\,dx=(ax-y^2)\,dy, (237)x\,dy-y\,dx=xy^2\,dx, (238)(1+xy)x\,dy+(1-xy)y\,dx=0, \\
(239)(y\,dx-x\,dy)+\log x\,dx &= 0, (240)x\,dy-y\,dx=x\cos^2\left(\frac{y}{x}\right)dx, (241)x^2y\,dx-(x^3+y^3)\,dy=0, (242)xy\,dx-(x^2+2y^2)\,dy=0, \\
(243)(3xy^2-y^3)\,dx-(2x^2y-xy^3)\,dy &= 0 (244)y(x^2y^2+2)\,dx+x(2-2x^2y^2)\,dy=0, (245)(x^3y^3+x^2y^2+xy+1)\,y\,dx-(x^3y^3-x^2y^2-xy-1)x\,dy=0 \\
(246)2xy\,dy-(x^2+y^2+1)\,dx &= 0, (247)\left(xy^2-e^{\frac{1}{x^3}}\right)dx-x^2y\,dy=0, (248)(x^3-2y^2)dx+2xy\,dy=0, (249)(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0, \\
(250)(1-x^2)y+2xy &= x\sqrt{1-x^2} (251)(1+y^2)x'=\tan^{-1}y-x\left(x'=\frac{dx}{dy}\right), (252)\frac{dy}{dx}+y\cot x=2\cos x, (253)\frac{dy}{dx}-y\tan x=-2\sin x, (x+y+1)\frac{dy}{dx}=1 \\
(254)(x+2y^3)\frac{dy}{dx} &= y, (255)\frac{dy}{dx}+\frac{y}{x}=y^2x, (256)\frac{dy}{dx}+\frac{xy}{1-x^2}=x\frac{1}{y^2}, (257)x\frac{dy}{dx}+y=y^2x^3\cos x, (258)\frac{dy}{dx}=e^{x-y}(e^x-e^y), (259)\frac{dy}{dx}=x^3y^3-xy
\end{aligned}$$

UNIT-III(HIGHER ORDER LINEAR D.E)

Solve the Differential Equation:

- (1) $\frac{d^3 y}{dx^3} - 9\frac{d^2 y}{dx^2} + 23\frac{dy}{dx} - 15y = 0$, (2) $\frac{d^3 x}{dt^3} - 2\frac{d^2 x}{dt^2} - 3\frac{dx}{dt} = 0$, (3) $\frac{d^4 y}{dx^4} + 13\frac{d^2 y}{dx^2} + 36y = 0$, (4) $\frac{d^4 x}{dt^4} + 4x = 0$, (5) $y'' - 2y' + 3y = 0$, $y(0) = 1$, $y'(0) = 0$,
 (6) $y''' - 5y'' + 7y' - 3y = 0$, $y(0) = 1$, $y'(0) = 0$, $y''(0) = -5$, (7) $(D^2 - 2D + 5)^2 y = 0$, (8) $y^{iv} + 32y'' + 256y = 0$, (9) $(D^3 + 1)^3 (D^2 + D + 1)^2 y = 0$,
 (10) $\frac{d^2 y}{dt^2} + 4\frac{dy}{dt} + 29y = 0$, when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 15$, (11) $(D^3 - 6D + 11D - 6)y = e^{-2x} + e^{-3x}$, (12) $(D - 2)^2 y = 8(e^{2x} + \sin 2x + x^2)$,
 (13) $(D + 2)(D - 1)^2 y = e^{-2x} + 2 \sin hx$, (14) $\frac{d^4 y}{dx^4} - y = \cos x \cosh x$, (15) $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$, (16) $(D^2 + 1)y = x^2 e^{3x}$, (17) $(D^3 + 1)y = 3 + 5e^x$,
 (18) $(D^2 + 4)y = e^x + \sin 2x + \cos 2x$, (19) $y'' + 4y' + 4y = 4 \cos x + 3 \sin x$, $y(0) = 1$, $y'(0) = 0$, (20) $y'' + 4y' + 20y = 23 \sin 3t - 15 \cos t$, $y(0) = 0$, $y'(0) = -1$,
 (21) $(D^2 + 4)y = \sin t + \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t$, $y(0) = 1$, $y'(0) = \frac{3}{35}$, (22) $D^2 (D^2 + 4) = 320(x^3 + 2x^2)$, (23) $(D^3 - D^2 - 6D)y = 1 + x^2$, (24) $y''' + 2y'' - y' - 2y = 1 - 4x^3$,
 (25) $(D^2 - 7D + 6)y = e^{2x}(1 + x)$, (26) $(D^4 + 1)y = e^x \cos x$, (27) $(D^2 - 3D + 2)y = xe^{3x} + \sin 2x + x^2 e^{-3x}$, (28) $(D^2 + 9)y = 8x^2 e^{2x} \sin 2x$,
 (29) $(D^2 - 4D + 4)y = x^2 \sin x + e^{2x} + 3$, (30) $y'' + 4y' + 20y = 23 \sin t - 15 \cos t$, $y(0) = 0$, $y'(0) = -1$, (31) $(D^3 + 1)y = \cos(2x - 1)$, (32) $(D^2 + 1)y = x$,
 (33) $(D^2 + 4)y = \tan 2x$, (34) $\frac{d^2 y}{dx^2} - y = \frac{2}{1 + e^x}$, (35) $y'' + 6y' + 9y = \frac{e^{-3x}}{x}$, (36) $y'' - 2y' + y = e^x \log x$, (37) $y'' + y = \operatorname{cosec} x$

VARIATION OF PARAMETERES METHOD

$$(51) (D^2 + 1)y = x, (52) (D^2 + 4)y = \tan 2x, (53) \frac{d^2 y}{dx^2} - y = \frac{2}{1 + e^x}, (54) y'' + 6y' + 9y = \frac{e^{-3x}}{x}, (56) y'' - 2y' + y = e^x \log x, (57) y'' + y = \operatorname{cosec} x$$
$$(58) y'' - 2xy' + 2y = e^x \tan x, (59) y'' + 4y' + 4y = e^{-2x} \sin x, (60) y''' - 6y'' + 11y' - 6y = e^{-x}, (61) y''' - 14y' = \sec 2x, (62) y''' - 6y'' + 12y' - 8y = \frac{e^{2x}}{x}$$

Method of Reduction of Order

$$(71) x^2 y'' + 4xy' + 2y = 0, y_1(x) = \frac{1}{x}, (72) y'' - y' + 6y = 0, y_1 = e^{3x}, (73) (x^2 - 1)y'' - 2xy' + 2y = 0, y_1 = x, (74) x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0, y_1 = x^{\frac{-1}{2}} \sin x$$
$$(75) (x - 2)y'' - xy' + 2y = 0, x \neq 2, y_1 = e^x$$

Using the method of variation of parameters and for given L.I solution then find P.I & G.S

$$(76) x^2 y'' + xy' - y = x^3, y_1 = x, y_2 = \frac{1}{x}, (77) x^2 y'' + xy' - 4y = x^2 \log x, y_1 = x^2, y_2 = \frac{1}{x^2}, (78) x^2 y'' - xy' + y = \frac{1}{x^4}, y_1(x) = x, y_2 = x \log x$$
$$(79) y'' + 4y' + 8y = 16e^{-2x} \operatorname{cosec}^2 2x, y_1 = e^{-2x} \cos 2x, y_2 = e^{-2x} \sin 2x$$

Euler-Cauchy's Equations and Legendre's Linear Equation

$$\begin{aligned}(81) x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y &= 10 \left(x + \frac{1}{x} \right), (82) x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x, (83) x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \log x \sin(\log x) \\ (84) x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y &= x^4, (85) \frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} = \frac{12 \log x}{x^2}, (86) x^2 \frac{d^3 y}{dx^3} - 4x \frac{d^2 y}{dx^2} + 6 \frac{dy}{dx} = 4, \\ (87) x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y &= x^2 + 2 \log x, (88) x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos(\log x), \\ (89) x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + y &= \frac{\log x \sin(\log x)}{x} + 1, (90) (x^2 D^2 + 3xD + 1)y = \frac{1}{(1-x)^2}, \\ (91) \left(D^2 + \frac{1}{x} D \right) y &= \frac{12 \log x}{x^2}, (92) (x^2 D^2 + 4xD + 6)y = (\log x)^2, \\ (93) (3x+2)^2 \frac{d^2 y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y &= 3x^2 + 4x + 1, (94) (2x+3)^2 \frac{d^2 y}{dx^2} - 2(2x+3) \frac{dy}{dx} - 12y = 6x, \\ (95) (1+2x)^2 \frac{d^2 y}{dx^2} - 6(1+2x) \frac{dy}{dx} + 16y &= 8(1+2x)^2, \\ (96) \left((1+x)^2 D^2 + (1+x)D + 1 \right) y &= \cos[\log(1+x)], (97) \left((1+x)^2 D^2 + (x+1)D \right) y = (2x+3)(2x+4), \\ (98) \left((2x-1)^3 D^3 + (2x-1)D - 2 \right) y &= x\end{aligned}$$

Simultaneous Linear Differential Equation with Constant Co-efficients

$$\begin{aligned}(121) \frac{dx}{dt} + 4x + 3y &= t, \frac{dy}{dt} + 2x + 5y = e^t, (122) \frac{d^2 x}{dt^2} + 4x + 5y = t^2, \frac{d^2 y}{dt^2} + 5x + 4y = t + 1, (123) t \frac{dx}{dt} + y = 0, x(1) = 1 \text{ and } t \frac{dy}{dt} + x = 0, y(-1) = 0, \\ (124) \frac{dx}{dt} &= 7x - y, \frac{dy}{dt} = 2x + 5y, (125) \frac{d^2 x}{dt^2} - \frac{dy}{dt} = 2x + 2t, \frac{dx}{dt} + 4 \frac{dy}{dt} = 3y, (126) \frac{d^2 x}{dt^2} + y = \sin t, \frac{d^2 y}{dt^2} + x = \cos t, \\ (127) (2D - 4)y_1 + (3D + 5)y_2 &= 3t + 2, (D - 2)y_1 + (D + 1)y_2 = t, (128) (2D + 3)y_1 + (D + 5)y_2 = t^2, (8D + 14)y_1 + (11D + 28)y_2 = c^{2t} \\ (129) (D + 3)y_1 + (3D + 23)y_2 &= e^{-2t}, (D + 2)y_1 + (4D + 14)y_2 = e^{2t}, (130)\end{aligned}$$

UNIT-IV(POWER SERIES SOLUTIONS AND GAMMA,BETA FUNCTIONS)

1. Classify the singular points of Differential Equations are given below

- (1) $(1-x^2)y'' - 2xy' + n(n+1)y = 0$, (2) $(1-x^2)y'' - 2xy' + 2y = 0$, (3) $x^2y'' + (x+x^2)y' - y = 0$,
 (4) $x^2y'' + xy' + (x^2 - n^2)y = 0$, (4) $x^2y'' + axy' + 6y = 0$, (5) $x^3(x-2)y'' + x^3y' + 6y = 0$, (6) $x^2y'' + \sin xy' + \cos xy = 0$,
 (7) $x^4y'' + 4x^3y' + y = 0$, (8) $x^3(x^2-1)y'' - x(x+1)y' - (x-1)y = 0$, (9) $x^2y'' + 4y' + (x^2+2)y = 0$,
 (10) $(x^2+x-2)^2y'' + 3(x+2)y' + (x-1)y = 0$, (11) $2x^2y'' + 7x(x+1)y' - 3y = 0$, (12) $(4+x^2)y'' - 6xy' + 8y = 0$,
 (13) $2x^2y'' + (2x^2-x)dy' + y = 0$, (14) $x(x+1)y' - (2x+1)y = 0$, (15) $2x^2y'' + xy' - (x^2+1)y = 0$,
 (16) $9x(1-x)y'' - 12y' + 4y = 0$, (17) $(x-x^2)y'' + (1-5x)y' - 4y = 0$, (18) $2x(1-x)y'' + (5-7x)y' - 3y = 0$

2. Find the series solution about the point $x = 0$, of the following Differential Equation.

- (21) $y' + 3y, y' - 4y = 0$, (22) $(1-x^2)y' = 2xy$, (23) $(x-1)y' = xy$, (24) $y' = xy$, (25) $(1+x)y' + xy = 0$, (26) $y'' + xy' - y = 0$, (27) $y'' + 4y = 0, y(2) = 2, y'(2) = 2$
 (28) $x(x+1)y' - (2x+1)y = 0$, (29) $xy' - (x+2)y = 0$, (30) $y'' - y = 0$, (31) $y'' + 4y = 0$, (32) $(1-x^2)y'' - 2xy' + 2y = 0$, (33) $y'' - xy = 0, y(1) = 2, y'(1) = 0$
 (34) $(2+x^2)y'' - 2xy' + 3y = 0, y(1) = 1, y'(1) = -1$, (35) $(1+2x)y'' - y' + y = 0, y(0) = 0, y'(0) = 1$, (36) $(1-x)y'' + xy' + 2y = 0, y(0) = 2, y'(0) = 1$,
 (37) $(2-x^2)y'' + 2xy' - 2y = 0$, (38) $(4+x^2)y'' - 6xy' + 8y = 0$, (39) $y'' + (x-1)^2y' - 4(x-1)y = 0$, (40) $y'' + 2xy' + y = 0$, (41) $(1-x^2)y'' - 2xy' + 6y = 0$,
 (42) $(1+x^2)y'' - 9y = 0$, (43) $(1-x^2)y'' + 2xy' + y = 0, y(0) = 1, y'(0) = 1$, (44) $(4+x^2)y'' - 6xy' + 8y = 0$, (45) $x(x+1)y'' - (2x+1)y = 0$,
 (46) $(1+x^2)y'' - 9y = 0$, (47) $y'' + t^2y' - 4ty = 0$, (48) $(x^2+2)y'' + xy' - (1+xy) = 0$, (49) $y'' - 2x^2y' + 4xy = x^2 + 2x + 4$,

3. Find the series solution about the indicated point $x = x_0$ of the following Differential Equation.

- (51) $y' = 2y, x_0 = 1$, (52) $y'' - y = 0, x_0 = 1$, (53) $y'' + xy' + y = 0, x_0 = 2$, (54) $(x+1)y' - (x+2)y = 0, x_0 = -2$
 (55) $xy' - y = 0, x_0 = 1$, (56) $y'' + (x-1)y' + y = 0$, at $x_0 = 2$, (57) $(1+2x)y'' + xy' + y = 0$, at $x_0 = 1, y(1) = 1, y'(1) = -1$
 (58) $y'' + xy' - y = 0, x_0 = 1$, (59) $(1+x)y'' - xy' - y = 0$, at $x_0 = 2, y(2) = 1, y'(2) = 0$

4. Find the series solution about the $x = 0$ point of the following Differential Equation by Frobenius method.

- (61) $2x^2y'' + xy' - (x^2+1)y = 0$, (62) $xy'' + y' - xy$, (63) $x(1+x)y'' + 3xy' + y = 0$, (64)
 $x^2y'' + 6xy' + (6+x^2)y = 0$ (65) $(1-x^2)y'' - 2xy' + 6y = 0$, (67) $2x^2y'' + (2x^2-x)y' + y = 0$, (68)
 $x^2y'' + xy' + (x^2-4)y = 0$ (69) $x(1+x)y'' + 3xy' + y = 0$ (70) $2x(1+x)y'' + (1+x)y' - 3y = 0$, (71)

$$4x^2y'' - 8xy' + 5y = 0, (72) \quad xy'' + (1 - 2x)y' + x(x - 1)y = 0, (73) \quad (x + x^2)y'' + (1 + x)y' - y = 0, (74)$$

$$x^2y'' + xy' + (x^2 - n^2)y = 0, (75) \quad 9x(1 + x)y'' - 6y' + 2y = 0$$

$$(76) \quad 2x(1 - x)y'' + (1 - x)y' + 3y = 0, (77) \quad 3x(1 - x)y'' + 2(1 - x)y' + 4y = 0, (78)$$

$$2x^2(1 + x^2)y'' - 3x(1 + x^2)y' + 2y = 0, (79) \quad xy'' + (1 - 2x)y' + (x - 1)y = 0, (80) \quad 4(x^2 + x^4)y'' + 16x^3y' + y = 0, (81)$$

$$2x^2y'' + xy' - 3y = 0, (82) \quad 16x^2y'' + 3y = 0$$

$$(83) \quad x^2y'' + 4xy' + (x^2 + 2)y = 0, (84) \quad x^2y'' + xy' + (x^2 - 1)y = 0, (85) \quad x^2y'' + 6xy' + (6 - 4x^2)y = 0$$

$$(86) \quad 9x(1 - x)y'' - 12y' + 4y = 0, (87) \quad 2x^2y'' + xy' - (x + 1)y = 0, (88) \quad 2x(1 - x)y'' + (1 - x)y' + 3y = 0$$

$$(89) \quad 2x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + (1 - x^2)y = x^2, (90) \quad 2x(1 - x)y'' + (5 - 7x)y' - 3y = 0, (91) \quad 2x^2y'' + (2x^2 - x)y' + y = 0,$$

$$(91) \quad x(x - 1)y'' + (3x - 1)y' + y = 0, (92) \quad (x - x^2)y'' + (1 - 5x)y' - 4y = 0, (93) \quad xy'' + (1 + x)y' + 2y = 0,$$

$$(94) \quad x^2y'' - x(1 + x)y' + y = 0, (95) \quad x^2y'' + xy' + (x^2 - y)y = 0, (96) \quad x(1 - x) \frac{d^2y}{dx^2} - (1 + 3x)y' - y = 0$$

5. Find the series solution about the indicated point $x = x_0$ of the following Differential Equation by Frobenius method.,

$$(101) \quad 2(1 - x)y'' - xy' + y = 0, x = 1, (102) \quad 9x(1 + x)y'' - 6y' + 2y = 0, x = -1, (103)$$

$$x(x - 2)y'' + 4y' + 3y = 0, x = 2$$

$$(104) \quad y'' + (x - 1)y' + y = 0, x_0 = 2$$